Wave Shaping

Waveshaping is a type of distortion synthesis that can be effective and efficient for creating dynamic and evolving spectra. In fact, we've already implemented a waveshaping algorithm in the form of the dynamic range compressor. In this section, we take a closer look at the use of memoryless, nonlinear functions applied to input waveforms to produce dynamic, amplitude-dependent spectra.

Basic Technique

- Waveshaping is simply the processing of an input waveform by a nonlinear input/output transfer function, as depicted in Fig. 1.

- **Figure 1:** The processing of a sinusoidal input by a waveshaping function of the form $f(x) = (x + x^3) / 2$.
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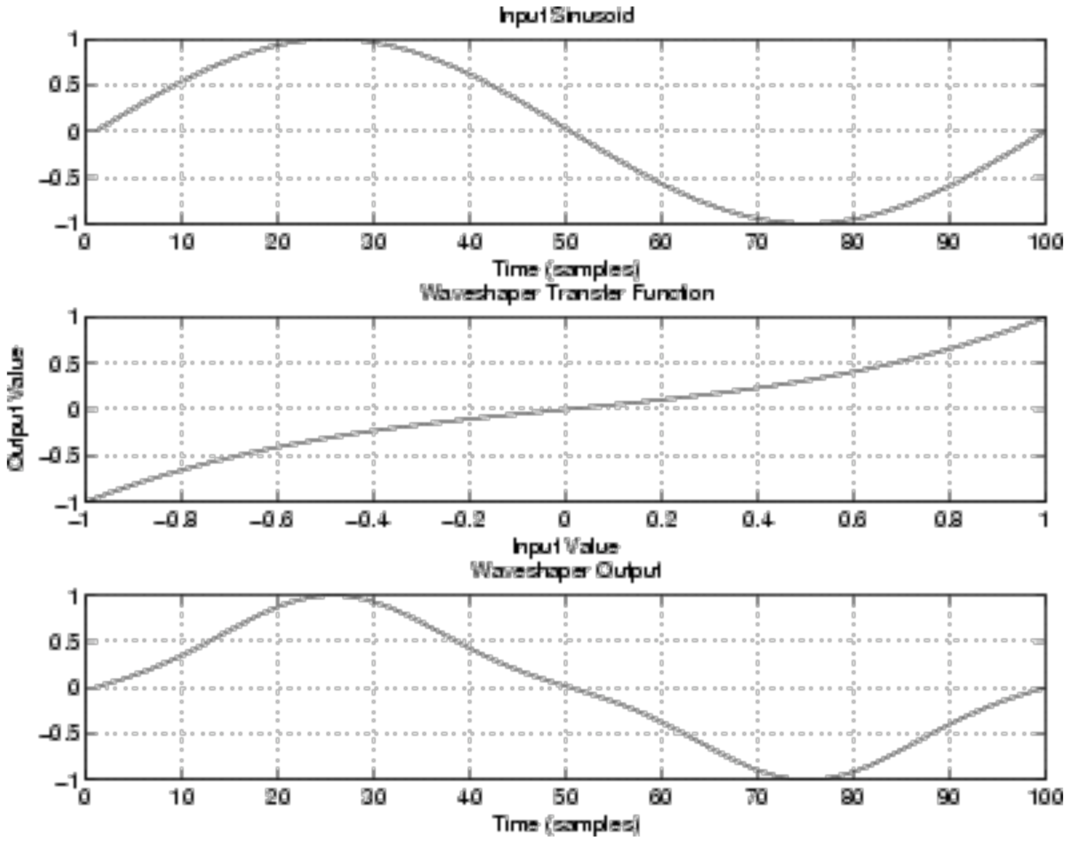
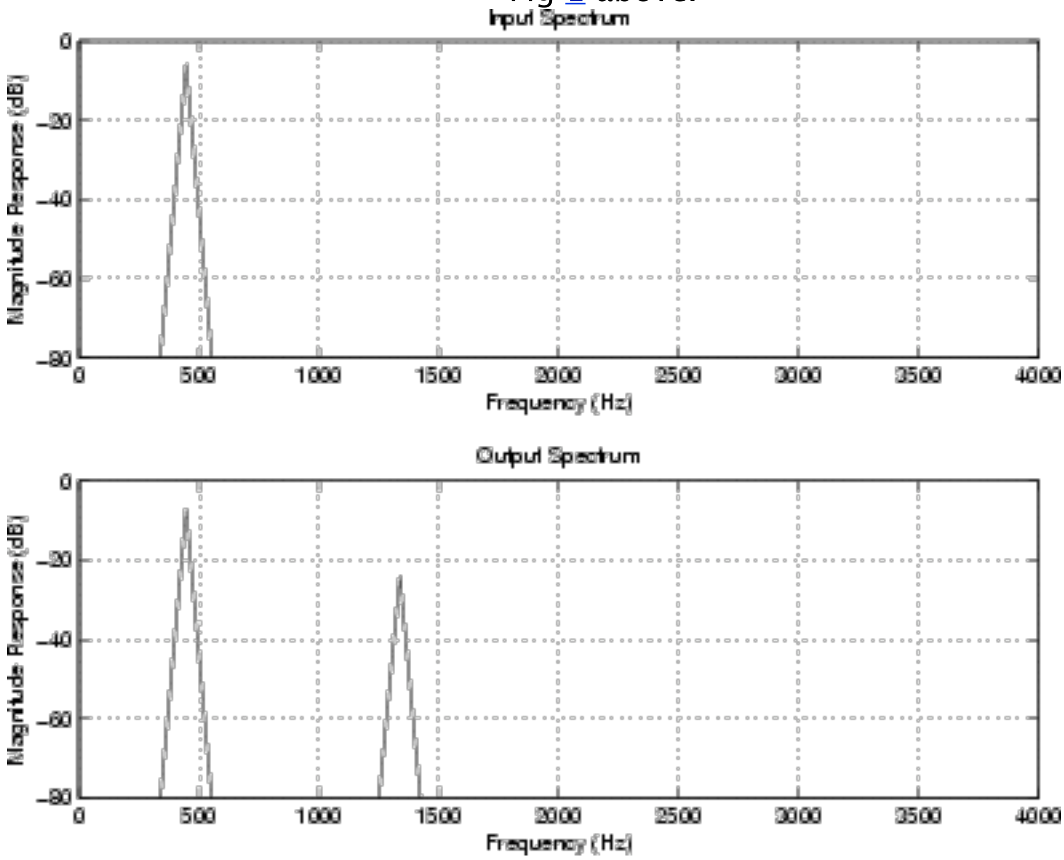


- In general, waveshaping transfer functions are only dependent on current values of time (i.e., the functions are ``memoryless'').

- In altering the input waveform, new spectral components are created where they didn't exist before, as shown in Fig. 2.

- **Figure 2:** Input and output spectra for the waveshaping process depicted in Fig 1 above.



- Linear systems do not change the spectral content of a signal. Thus, this process is inherently non-linear.

Polynomial Transfer Functions

- Under certain limited circumstances, the waveshaping operation can be described analytically.
- In particular, when a waveshaping transfer function is expressed in terms of a polynomial, it is relatively simple to determine the resulting output signal spectrum weighting given a sinusoidal input of amplitude 1.
- A generic waveshaping polynomial transfer function can be expressed as:

$$f(x) = d_0 + d_1 x + d_2 x^2 + \ldots + d_N x^N$$

- Also under these conditions, no spectral energy is produced at frequencies higher than the highest exponent value times the input signal frequency.

- Odd powers of the waveshaping function produce odd harmonic distortion. Even powers of the waveshaping function produce even harmonic distortion.
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- For example, if we process the sinusoid from Fig. 1 with a waveshaping function of the form $f(x) = x + x^2 + x^3$, we get a resulting spectra as shown in Fig. 3.
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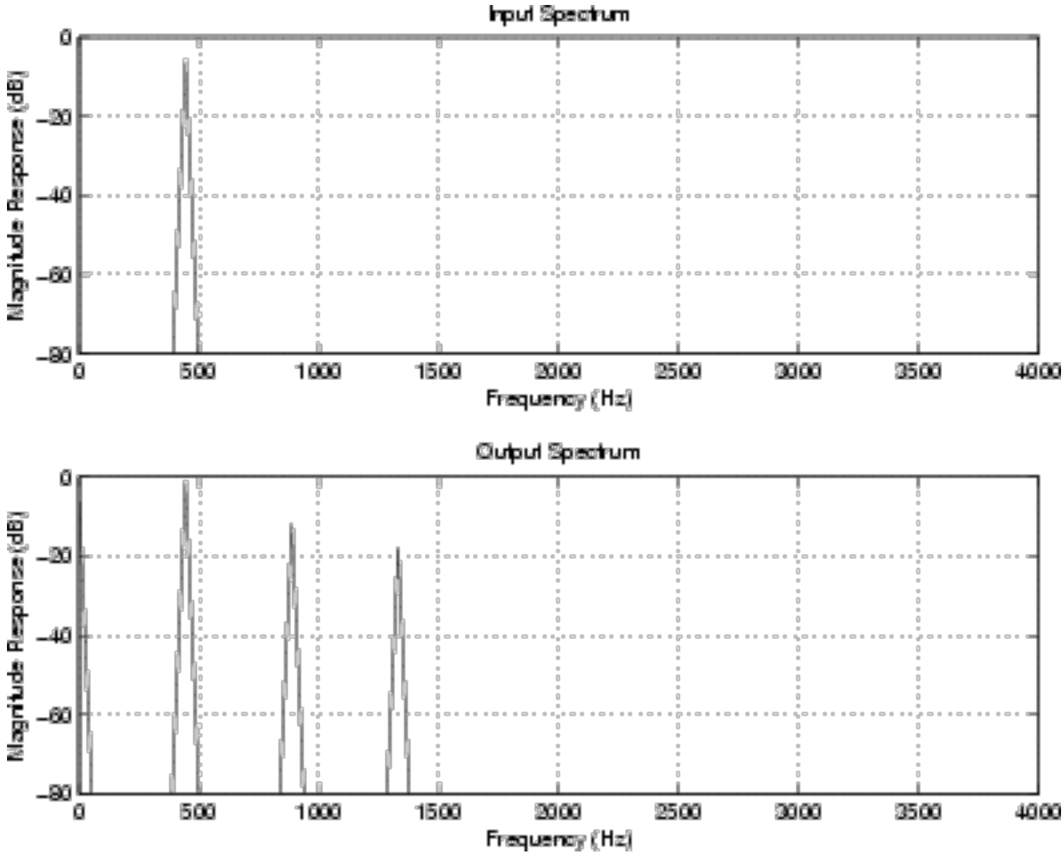
- **Figure 3:** Input and output spectra for a waveshaping function of the form $f(x) = x + x^2 + x^3$.



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- We can also think of waveshaping with polynomials as a form of modulation synthesis. In the above example, when the sinusoidal signal (of frequency f_1) is squared, we form sum and different components at 0 and $2f_1$ Hz. When the squared signal is again multiplied by the original sinusoid, we also get components at f_1 and $3f_1$.

Amplitude Dependence

- When the sinusoidal input amplitude is given by a (instead of 1), the output is then scaled by a as:
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- $$f(ax) = d_0 + d_1 ax + d_2 a^2 x^2 + \ldots + d_N a^N x^N$$
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- Because higher values of a generally produce richer spectra, a is referred to as the distortion index.
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- Because the results of waveshaping with complex input signals are difficult to predict, most waveshaping applications use sinusoidal input signals only.
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- When a sinusoid of unity amplitude is applied to a Chebyshev polynomial of order k, the output contains energy only at the kth harmonic. This property makes Chebyshev polynomials potentially useful for building more complex waveshaping functions in terms of a specific desired harmonic content.
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- Waveshaping systems will often incorporate output scaling operations to compensate for excessive gains that result from the waveshaping function.